2.1

- $\frac{\partial}{\partial x}\left[\ln \frac{x}{y}\right] = \frac{\partial}{\partial x}\left[\ln x - \ln y\right] = \frac{\partial}{\partial x} \ln x = \frac{1}{x}$

- $\frac{\partial}{\partial v}\left[\frac{nRT}{v}\right] = nRT \frac{\partial}{\partial v}\left[\frac{1}{v}\right] = nRT \frac{\partial}{\partial v}\left[v^{-1}\right] =$

$= nRT\,(-1) \cdot v^{-2} = -\frac{nRT}{v^2}$

2.2

$\vec{\nabla} f = (\partial_x, \partial_y, \partial_z) f$

$f(x,y,z) = \log \frac{\sqrt{x}}{yz} + \arcsin xz$

$$\begin{cases} \partial_x f = \frac{1}{2}\frac{1}{x} + \frac{z}{\sqrt{1-(xz)^2}} \\ \partial_y f = -\frac{1}{y} \\ \partial_z f = -\frac{1}{z} + \frac{x}{\sqrt{1-(xz)^2}} \end{cases}$$

$$\vec{\nabla} f = \left(\frac{1}{2x} + \frac{z}{\sqrt{1-(xz)^2}} , -\frac{1}{y} , -\frac{1}{z} + \frac{x}{\sqrt{1-(xz)^2}} \right)$$

2.3

$$T(x,y,z) = e^{-x^2-2y^2-3z^2}$$

$$p = (1,1,1)$$

$$-\vec{\nabla} T\big|_p = -(\partial_x, \partial_y, \partial_z)\, T\big|_p$$

$$\begin{cases} \partial_x T\big|_p = e^{-x^2-2y^2-3z^2} \cdot (-2x)\Big|_{p=(1,1,1)} = \\ \quad = e^{-6}(-2) \\ \partial_y T\big|_p = e^{-x^2-2y^2-3z^2} \cdot (-2\cdot 2\cdot y)\Big|_{p=(1,1,1)} = \\ \quad = e^{-6}(-4) \\ \partial_z T\big|_p = e^{-x^2-2y^2-3z^2} \cdot (-3\cdot 2\cdot \cancel{y} z)\Big|_{p=(1,1,1)} = \end{cases}$$

$$= e^{-6} \cdot (-6)$$

$$-\overline{\nabla} T\big|_p = \cancel{-}e^{-6}(-2,-4,-6) =$$

↑ vector that points the direction of travel to decrease the T as fast as possible

$$= 2e^{-6}(1,2,3)$$

2.4

$$g(x,y) = x^2 - y^2 \qquad P = (2,3)$$

$$-\bar{\nabla} g|_P = -(\partial_x, \partial_y) g|_P$$

$$\partial_x g|_P = 2x|_{P=(2,3)} = 2\cdot 2 = 4$$

$$\partial_y g|_P = -2y|_{P=(2,3)} = -2\cdot 3 = -6$$

$$-\bar{\nabla} g|_P = -(4,-6)$$

← vector that point the direction of the bug

lets find the unitary vector (just for fun) not required in this exercise

$$\bar{v} = (v_x, v_y)$$

$$\hat{v} = \frac{(v_x, v_y)}{\sqrt{v_x^2 + v_y^2}}$$

$$\bar{v} = -(4,-6)$$

$$\hat{v} = \frac{-(4,-6)}{\sqrt{4^2+6^2}} = \frac{-1}{2\sqrt{13}}(4,-6) = \frac{-\cancel{2}}{\cancel{2}\sqrt{13}}(2,-3)$$

$$= \frac{-1}{\sqrt{13}}(2,-3)$$

(35) Do yourself, you can use geogebra ...

(36) $f(x,y,z) = x^2 - y^2 + xyz^3 - zx$

$P = (1,2,3)$

a) Directional derivative $\hat{u} = \frac{1}{\sqrt{2}}(1,-1,0)$

$$\overset{\shortparallel}{\nabla f|_p \cdot \hat{u}} = *$$

lets compute $\nabla f|_p$ first

$$\nabla f|_p = (\partial_x, \partial_y, \partial_z) f|_p = (53, 23, 53)$$

$$\begin{cases} \partial_x f = 2x + yz^3 - z|_p = 2\cdot 1 + 2\cdot 3^3 - 3 = 53 \\ \partial_y f = -2y + xz^3|_p = -2\cdot 2 + 1\cdot 3^3 = 23 \\ \partial_z f = 3xyz^2 - x|_p = 3\cdot 1\cdot 2\cdot 3^2 - 1 = 53 \end{cases}$$

$$* = (53, 23, 53)\cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(53\cdot 1 + 23\cdot(-1) + 53\cdot 0\right)$$

$$= \frac{30}{\sqrt{2}} = 21{,}21$$

CEU
Universidad
San Pablo

ESCUELA POLITÉCNICA SUPERIOR

CARRERA				
APELLIDOS	NOMBRE		GRUPO	CALIFICACIÓN
ASIGNATURA	FECHA	DNI		

b) direction of $\overline{\nabla} S|_P = (53, 23, 53)$

is the one where directional derivative

take the maximum value

the value is : $|\overline{\nabla} S|_P| = \sqrt{53^2 + 23^2 + 53^2}$

$= 78,4$

max value of
directional derivative

(2.7)

$$f(x,y) = \frac{x^2-y^2}{x^2+y^2} \qquad P=(1,1)$$

direction such as directional derivative vanish

$\hat{v}$ $\quad$ tg $\quad$ $\vec{\nabla} f|_P \cdot \hat{v} = 0$

$$\overline{\nabla} f|_P = (\partial_x, \partial_y) f|_P =$$

$$\partial_x f|_P = \frac{(x^2-y^2)' \cdot (x^2+y^2) - (x^2-y^2)\cdot(x^2+y^2)'}{(x^2+y^2)^2} =$$

$$= \left.\frac{2x\,(x^2+y^2) - (x^2-y^2)\,2x}{(x^2+y^2)^2}\right|_{P=(1,1)} =$$

$$= \frac{2(2) - 0\cdot 2}{2^2} = \frac{4}{4} = 1$$

$$\partial_y f|_P = \cancel{\ldots} = \left.\frac{-2y(x^2+y^2) - (x^2-y^2)\,2y}{(x^2+y^2)^2}\right|_{P=(1,1)} =$$

$$= \frac{-2\cdot 2 - 0}{2^2} = \frac{-4}{4} = -1$$

$$\overline{\nabla} f|_P = (1,-1) \qquad \hat{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\begin{cases} \nabla g|_P \cdot \hat{v} = 0 \\ |\hat{v}| = 1 \end{cases} \qquad \begin{cases} (1, -1) \cdot \begin{pmatrix} v_x \\ v_y \end{pmatrix} = 0 \\ \sqrt{v_x^2 + v_y^2} = 1 \end{cases}$$

$$\begin{cases} v_x - v_y = 0 \\ v_x^2 + v_y^2 = 1 \end{cases} \Rightarrow \begin{cases} v_x = \frac{1}{\sqrt{2}} \\ v_y = \frac{1}{\sqrt{2}} \end{cases}$$